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# **MILITARY COMPENSATION AND RETENTION: AN ANALYSIS OF ALTERNATIVE MODELS AND A SIMULATION OF A NEW RETENTION MODEL**

John T. Warner

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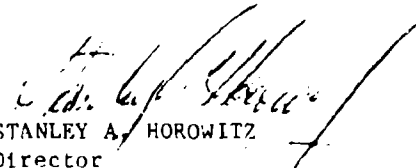
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# **MILITARY COMPENSATION AND RETENTION: AN ANALYSIS OF ALTERNATIVE MODELS AND A SIMULATION OF A NEW RETENTION MODEL**

John T. Warner



*Institute of Naval Studies*

**CENTER FOR NAVAL ANALYSES**

2000 North Beauregard Street, Alexandria, Virginia 22311

#### ABSTRACT

Four major models for predicting the effects of changes in military pay on retention are described and compared. The most sophisticated model, called the Stochastic Cost of Leaving or SCOL model, is simulated to demonstrate the effects of several changes in military compensation on retention.

## EXECUTIVE SUMMARY

This paper has been written to accomplish two tasks. The first of these is to provide a comprehensive comparative analysis of the various models which have been or are now being used to predict the effect on retention of changes in military pay. The second task is to construct and execute a simulation of the latest model. To the extent that this model accurately reflects retention behavior in the Navy, this simulation demonstrates the effect on retention of several changes in military compensation.

In chronological order of their development, the models to be examined are the PVCOL (Present Value of the Cost of Leaving) Model, the ACOL (Annualized Cost of Leaving) Model, the SCOL (Stochastic Cost of Leaving) model, and the Air Force--Congressional Budget Office model. All of these models develop some measure of the difference between the income stream from staying in service at least one more term and the income stream from leaving now. This difference is called the cost of leaving. Streams of future income are converted to single numbers by discounting them by the rate at which personnel appear to discount future income. The cost of leaving is then related to the retention rate via some supply function.

The PVCOL model calculates the cost of leaving as the maximum present value of the income stream from staying minus the present value of the income stream from leaving now. The retention rate is related to this cost of leaving via a logistic supply function, the parameters of which are estimated by regression analysis.

This model is deficient in several respects. First, it does not incorporate an individual's taste for military service, only monetary values. That is, the model focuses attention on individuals who are "taste neutral," not individuals who are actually on the margin of a stay-leave decision. Second, the model only looks at the effect of future pay changes on current behavior. It ignores the effect of past compensation practices on the current population eligible for reenlistment, a relationship that becomes important in the analysis of alternative retirement systems as well as other changes to the compensation systems (e.g., higher bonuses). Third, the connection between the cost of leaving and retention, the logistic function, is ad hoc. Fourth, it assumes the civilian and military income streams are known with certainty. Finally, some of the predictions from the model seem unreasonable.



The ACOL model remedies some of these criticisms, in whole or in part. First, the model introduces a taste for service factor into the stay-leave decision. This taste factor is the annual military-civilian pay differential required to make an individual indifferent between military and civilian life. When this taste factor is added, the model directs attention to those who are actually on the margin of a reenlistment decision, not those who are "taste neutral." The model derives the annualized cost of leaving (ACOL), which is the maximum annualized military-civilian pay differential from remaining in military service. The time horizon, or years of future military service, relevant for retention decision-making is the horizon over which ACOL is maximized. The retention rate at a given term of service will be the proportion of individuals for whom the maximum annual pay differential, or ACOL, exceeds the taste factor, or differential required to make them indifferent between military and civilian life.

A crucial input into the ACOL model is the assumed pattern of personal discount rates. Empirical analysis by Gilman (reference 1) suggests that young personnel (e.g., first-term personnel) have discount rates of about 20 percent per year, but that discount rates decline as individuals age. The assumed discount rate is crucial in the ACOL model because an increase in the discount rate serves to raise ACOL values calculated over short time horizons relative to those calculated over longer time horizons and, therefore, to reduce the time horizon that is relevant for retention decision-making. The assumed discount rate will thus have consequences for the predicted effect of a pay change.

To provide examples, if one assumes 20 percent discount rates among first-term reenlistment eligibles, the ACOL model predicts that the time horizon relevant for reenlistment decision-making is the length of a reenlistment. At a 10 percent discount rate, the maximum ACOL may be the one over the horizon that encompasses the 20th year of service. At the second-term reenlistment decision-point, the situation is different. Regardless of the discount rate, the time horizon relevant for reenlistment decision-making invariably encompasses the 20th year of service.

Note a strong implication of this model. That is that pay changes that occur past the time horizon over which ACOL is maximized will have no effect on retention rates. Thus, if the time horizon of first-term reenlistees is only the length of a reenlistment, a cut in retirement benefits will not hurt first-term retention. Conversely, an increase in pay beyond the second term may not raise first-term retention, unless it alters the time horizon over which ACOL is maximized.

In the ACOL model, each individual is presumed to know his future military and civilian income streams with certainty and he is implicitly unaware that future events may induce him to leave. The SCOL model departs from the ACOL model by adding a "transitory" or random disturbance to the retention decision at each term of service. This disturbance summarizes all of the influences on the individual's retention decision at each term of service that are not already reflected in his taste for service factor. While the individual cannot know future values of these transitory or random disturbances, it is assumed that he knows the probability that a random event will induce him to leave at each future term of service. The individual is then assumed to calculate his cost of leaving, based not on a single future horizon of military service, but rather on a weighted average of his leaving costs over all possible future terms of service, where the weights in this calculation are his perceived probabilities of leaving after each possible future horizon of service. The cost of leaving thus becomes a "stochastic" cost of leaving (hence the acronym SCOL).

Note that this model alters some of the conclusions of the other models. First, unlike the ACOL model, pay changes that occur in any future term of service will have some effect on current retention, albeit however small. There is always some probability that an individual will stay in service long enough to get that higher pay. Hence, this model will predict some effect of a reduction in 20-year retirement benefits on first-term retention; under certain assumptions stated above the ACOL model will not.

Second, far term pay changes have a smaller effect on the current term retention rate in the SCOL model than the PVCOL model. This is because the model accounts for the fact that individuals, even those with strong positive tastes for military service, know that there is some probability that they will not stay in service long enough to be affected by the far-term pay change. The probabilities of leaving after each term of service essentially operate as an extra discounting factor.

The addition of a transitory disturbance also allows a retention function to be derived from the model. This retention function has the property that as pay in past terms increases and past term retention increases, the retention rate in the current term decreases. This specification of the retention function makes rigorous the link between the current term retention rate and past compensation policies.

While the SCOL model provides several analytical advances over previous models, its disadvantages should be noted. First, it is

considerably more complicated mathematically than previous models. This increased complexity will no doubt provide a barrier to understanding among some users and it may inhibit the correct interpretation of the model's results.

Second, the model is a steady-state model. Beginning with a cohort of first-term eligibles characterized by some initial taste distribution, the SCOL model determines what fraction of this cohort will survive to each term of service under alternative pay regimes. By steady-state we mean that each successive cohort of reenlistment eligibles is characterized by the same taste distribution and that there are no unanticipated changes in the compensation system. Because the model is a steady-state model, it is not well suited to dynamic policy analysis. That is, the model cannot be used, say, to predict the effect on the Navy enlisted force over the next several years of a 10 percent pay increase. Rather, the SCOL model may only be used to compare the steady-state force that would evolve with a given pay change to the "base case" force, i.e., the steady-state force that would exist under the current compensation system. The great advantage of the ACOL model is that it can be used in dynamic forecasting.

A third difficulty is that empirical estimation of the model's parameters requires an estimation technique that is considerably more difficult than, say, regression analysis. Empirically, the parameters to be estimated are those of the initial taste distribution (distribution of tastes among the first-term reenlistment eligibles) and the transitory disturbance distribution. Estimation of these parameters requires longitudinal data on a cohort that has made at least two retention decisions. The estimation procedure is essentially "plug and chug" maximum likelihood--the parameters are varied until the best fit to the observed data is obtained. Unlike regression analysis, holding constant other systematic factors that affect retention (e.g., education, mental group, race) would be extremely difficult.

The Air Force-CBO model is similar to the SCOL model in that it too calculates a cost of leaving that is a weighted average of leaving costs over various possible future horizons of military service. The major difference is that the cost of leaving calculations are made on the basis of the probability of surviving to each future term of service and then leaving of those who in fact stay. This cost of leaving is again related to retention rates via a logistic supply function. The main criticism of this model is that it probably will lead to overprediction of the effect of changes in far future term pay and underpredict the effect of near term pay. The model uses survival rates to each future

term of those who in fact stay because these rates are observable. Yet, if those who left had in fact stayed, they would have had lower survival rates. Hence, the average survival rates used by the model are too high.

I now turn to simulation results from the SCOL model. To do the simulations, we first fit the model's parameters such that at FY 1979 values of Navy pay and civilian pay the model predicted, as closely as possible, FY 1979 Navy enlisted retention rates by term of service. Note that this procedure treats the observed FY 1979 retention pattern as a steady-state pattern although it is not. We did what we did only to give the model some link to the actual Navy retention pattern by term of service. In any event, the purpose of the simulations is to show SCOL model predictions of the effects of various pay changes relative to the model's own base case, not to empirically estimate the model's parameters.

Once the model's parameters were chosen, we simulated several changes in active duty military pay to determine what the pay elasticities implied by the model. Our purpose in doing this was to determine how consistent the model is with previously estimated pay elasticities. Then, we simulated the model for two alternative retirement systems, the OSD retirement plan without early withdrawal privileges and the OSD retirement plan with early withdrawal privileges. (See reference 4 for a detailed description of these plans.)

One reviewer noted that these simulations, by themselves, may not be very instructive because there are many combinations of model parameters that might generate the base case retention pattern, and the pay elasticities and estimates of the effects of alternative retirement systems might be very different for different parameter values. Note first that the range of values of the model's parameters such that the SCOL model will predict the current pattern of Navy enlisted retention (if not its actual values) is not large. Second, sensitivity analyses were performed for various parameter values, and the pay elasticities and estimates of effects of alternative retirement systems were quite stable for reasonable variations in the parameter values.

In our simulations, we find the following: First, a 10 percent increase in second-term pay elicits a 24.2 percent increase in the first-term retention rate. The implied pay elasticity of 2.42 is consistent with previous results. The predicted effect of a second-term pay change is quite consistent with that predicted by the ACOL model. Second, a 10 percent increase in the whole military pay table is

predicted to generate a 32.2 percent increase in the first-term retention rate--an elasticity of 3.22. The effect of a change in the whole pay table is larger than just a change in second-term pay. (When the time horizon of people on the margin of a retention decision is short (e.g., the length of a reenlistment), the ACOL model will predict no difference in the impact of short and long-term pay changes.) Yet, while the SCOL model does predict a difference, the model avoids the implausibly high predicted change (about 50 percent) provided by the PVCOL model. Third, in the SCOL model, higher pay serves to raise retention only over the interval during which the pay raise is in effect. Most of the personnel induced to stay by the higher pay leave thereafter.

In this simulation, the second-term retention rate is very responsive to third-term pay changes, considerably more responsive than estimates found elsewhere (reference 7). This high responsiveness is due to declining variation in the taste distribution as those with low tastes leave at the end of the first term. While increased responsiveness of retention rates at the second and later terms is expected on theoretical grounds, the inconsistency between these and other results requires further investigation.

SCOL model simulations of alternative retirement plans provide estimates that are reasonably consistent with those from the ACOL model (reference 4). Both models predict that the OSD plan without early withdrawal privileges would get lower retention rates prior to YOS 20, but higher rates thereafter. The main difference is that the SCOL model predicts a smaller change in second-term retention than does the ACOL model, but a larger drop in third-term retention. Both models predict about the same cumulative survival probability to YOS 20. Both models predict that the OSD plan with early withdrawal will generate higher retention prior to YOS 10 than the current system. Significantly, the predicted increase is larger than that obtained earlier using the ACOL model (reference 4). However, the predicted decline after YOS 10 is larger also. Overall, the SCOL model gives more optimistic estimates of the effect of the OSD plan than the ACOL model.

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## INTRODUCTION

During the past three years, much attention has been devoted to developing models that can predict the effect of changes in military compensation on personnel retention. Prior to 1978, virtually no models existed that could be used to predict the effect of such changes. Yet, beginning with the report of the President's Commission on military Compensation (PCMC) in 1978, several proposals for overhauling the military retirement system have surfaced. The PCMC plan, as well as one recently submitted to Congress by the Office of the Secretary of Defense (OSD), recommend that the current 20-year retirement system be replaced by a system that provides lower benefits to 20-year retirees, but significant cash benefits to those who complete 10 years of service. These proposals for overhaul of the retirement system and the likelihood of continuing pressure to alter other parts of the compensation system have emphasized the need to model the link between all forms of compensation and retention.

Subsequent to the deliberations of the PCMC, four retention models have been developed. The first is the PVCOL (present value of cost of leaving) model. This model was initially described by Gotz and McCall (reference 2) and later used by Warner (reference 3) to analyze the retention effects of the PCMC retirement plan. A variant of the PVCOL model is the ACOL (annualized cost of leaving) model. This model was developed by Warner (reference 4) and used to analyze alternative retirement systems. The third model is a variant of the PVCOL model and was first developed by Gotz and McCall (reference 5). I will call it the SCOL (stochastic cost of leaving) model, since the cost of leaving calculated by the model is a probabilistic, weighted average of leaving costs over various future time horizons (the weights represent the probabilities of existing at each of the possible future points). The fourth model, developed by the Air Force and the Congressional Budget Office, contains features of each of the first three models.

Much confusion has persisted about the differences in these models. Chipman (reference 6), for instance, describes the models and compares their predictions for the PCMC retirement plan, but he does not provide an analytical discussion of the differences in the models. The first objective of this paper, therefore, is to describe these models and demonstrate exactly how they differ. The conditions under which the various models predict (roughly) the same retention response to a pay change are derived, as well as the conditions under which the predicted responses will vary. The various models are compared and analyzed in

the next section. The second objective of this paper is to present some simulation results from the SCOL model. These results are compared with results from other models. This analysis is provided in the last section.

## THE MODELS

Before describing the models, let me make two points about retention patterns. First, there should be a natural tendency for retention rates to rise with term of service ( $t$ ). This tendency is separate and distinct from any increase in the financial incentive to stay and is due to the fact that in early terms of service the retention decision-making process serves to sort out those who like military service from those who don't. As this sorting process proceeds, the cohorts of personnel who stay will be comprised of people who, on average, have a higher taste for military service and hence higher retention rates.

The second point is that the retention rate at term  $t$  may not be independent of past pay policies. For example, suppose one group of first-term reenlistees receives a reenlistment bonus and a second (otherwise identical) group does not. The first-term reenlistment rate of the group receiving the bonus should be higher. However, at the second-term reenlistment point, the group receiving the first-term bonus may have a lower reenlistment rate (assuming both groups face the future military and civilian pay streams). This is because the group receiving the first-term bonus has a lower average taste for service at the second-term point than the group not receiving the bonus. The various models differ both in their ability to "explain" the natural tendency for retention rates to rise with term of service, and in their methodology for linking current term retention with past compensation policies. The major advance of the SCOL model is that both of the above phenomena are explained within (i.e. are "endogenous" to) the model.

We shall analyze the models in the following order: PVCOL, ACOL, SCOL, and the Air Force and CBO models. These models differ according to how (1) the error term in the model is specified, (2) the cost of leaving is calculated, and (3) the function relating the cost of leaving to the retention rate is specified. These differences may all lead to considerable differences in prediction of how personnel would respond to a given pay change.

### THE PVCOL MODEL

The original PVCOL model gave no consideration to non-monetary factors. By not explicitly considering non-monetary factors, the model could not explain why two individuals with equal costs of leaving might make different stay-leave decisions. Hence there was nothing in the

model to explain each individual's stay-leave decision. The significance of the fact that the model had no taste factor in it to explain the stay-leave decision should become evident in the discussion of this model, and the more complex models that follow.

One way to develop the PVCOL model is to consider an individual at the end of length of service (LOS)  $t$ . This individual can stay one more term and then leave, two more terms and then leave, etc.\* If  $T$  is the total number of terms of military service, the individual must evaluate  $T-t$  possible future income streams plus the income stream from leaving now. Denoting each of the possible future terms of service after which the individual may leave by the variable  $n$ , the return to staying from term  $t$  to term  $n$  may be defined as,

$$S_{t,n} = \sum_{j=t+1}^n M_j \beta^{j-t-1} + \beta^{j-t} (R_n + W_n) \quad (1a)$$

where:

$M_j$  = active duty military pay during term  $j$ ,  $j=1 \dots n$

$R_n$  = the present value at the end of term  $n$  of future retirement benefits vested after term  $n$

$W_n$  = the expected present value at the end of term  $n$  of future civilian earnings

$\beta = \frac{1}{1+d}$ , where  $d$  equals the individuals' rate of time preference or discount rate.

The return to staying from the end of term  $t$  to the end of term  $n$  is thus equal to the present value of the stream of active duty military pay from term  $t+1$  to the end of term  $n$  plus the present value at the end of term  $t$  of the streams of retired pay and civilian earnings the individual expects to receive if he waits until the end of term  $n$  to leave. The present value of the income stream from leaving now ( $I_t$ ) is simply the present value of the civilian future income stream the

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\* I say term of service rather than year of service because enlisted personnel typically have to make multiyear retention decisions. For example, to get a reenlistment bonus, personnel must reenlist for a minimum of three years.

individual expects to receive if he leaves now ( $W_t$ ) plus the present value of already vested retirement benefits ( $R_t$ ):

$$L_t = W_t + R_t \quad (1b)$$

The cost of leaving at the end of term  $t$  rather than remaining until the end of term  $n$  to leave ( $C_{t,n}$ ) is simply

$$C_{t,n} = S_{t,n} - L_t \quad (1c)$$

We may calculate  $T-t$  possible values of  $C_{t,n}$ . Which one is the one relevant for retention decision-making? The PVCOL model presumes that it is the maximum of these values. That is, this model assumes that the individual looks at the cost of leaving over various possible future horizons of military service (there are  $T-t$  of them) and bases his retention decision on the maximum value of  $C_{t,n}$ . We label the maximum of these values  $C_t$ .  $C_t$  represents the opportunity cost of leaving now rather than staying in for the highest future income stream to be had by additional military service.

An alternative approach to calculating  $C_t$  is via a recursive formulation. Beginning at the end of term  $T-1$ , we may calculate the return to staying  $S_{T-1}$  as  $M_t + \beta L_t$  and the return to leaving  $L_{T-1}$  as  $W_{T-1} + R_{T-1}$ . The cost of leaving,  $C_{T-1}$ , is  $S_{T-1} - L_{T-1}$ . The value of the optimal choice,  $V_{T-1}$ , is the maximum of  $S_{T-1}$  and  $L_{T-1}$ . The model is solved recursively for  $S_{T-2}$  and  $C_{T-2}$  where  $S_{T-2} = M_{T-1} + \beta V_{T-1}$ , and likewise for earlier years. The recursive formulation may be preferred over the "forward" formulation discussed above for computational reasons.

The PVCOL model relates  $C_t$  to the retention rate  $r_t$  via a logistic supply function:

$$r_t = \frac{1}{1 + e^{-(b_0 + b_1 C_t)}} \quad (2a)$$

or,

$$\ln \frac{r_t}{1-r_t} = b_0 + b_1 C_t \quad (2b)$$

This well-known function yields an S-shaped relationship between  $r_t$  and  $C_t$ , or a linear relationship between  $\ln \frac{r_t}{1-r_t}$  and  $C_t$ .

From either equation, we may derive the effect on  $r_t$  of changes in the future military pay stream. To do this, define  $n^*$  as the future term of service such that  $C_{t,n}$  is a maximum. To begin with, pay changes that occur past term  $n^*$  can have no effect on  $C_t$  and will hence be predicted to have no effect on  $r_t$  (unless  $n^*$  is shifted outward by the pay change). A pay change that occurs in the next term of service may be derived as follows. Since  $\frac{\partial C_t}{\partial M_{t+1}} = 1$ ,  $\frac{\partial r_t}{\partial M_{t+1}} = \frac{\partial r_t}{\partial C_t} \frac{\partial C_t}{\partial M_{t+1}} = b_1 r_t (1 - r_t)$ .

Consider now a pay change in term  $t+2$ . Since  $\frac{\partial C_t}{\partial M_{t+2}} = \beta$ ,

$$\frac{\partial r_t}{\partial M_{t+2}} = \frac{\partial r_t}{\partial C_t} \cdot \frac{\partial C_t}{\partial M_{t+2}} = \beta b_1 r_t (1 - r_t). \text{ Thus, a pay change that occurs in}$$

term  $t+2$  has a smaller effect on  $r_t$  than a pay change in term  $t+1$ , but they differ only by the discounting factor  $\beta$ .

Consider now the effect of an increase in the whole military pay table. The effect of  $r_t$  of a one dollar increase in the whole pay table is  $\left( \sum_{j=t+1}^{n^*} \beta^{j-t-1} \right) b_1 \cdot r_t (1 - r_t)$ . If term  $n^*$  is very many terms of service beyond term  $t$ , the (predicted) effect of this pay change will be large, in some cases unbelievably large. For example, the model predicted a 20 to 30 percent increase in first-term retention due to a 10-percent increase in second-term pay, which is in line with previous estimates. Yet, the model predicted a 50 to 60 percent increase in first-term retention due to a 10 percent increase in the whole pay table, and this appeared wholly unreasonable.\* The next two models predict smaller effect of a change in the whole pay table, but under certain conditions they will give estimates of a change in  $M_{t+1}$  similar to those obtained with the PVCOL model.

This PVCOL model is deficient in several respects. The first has already been alluded to--its estimates of the effect of an increase in the whole pay table seem too high. Second,  $r_t$  is not related to past pay, only future pay. The model is forward-looking only. Third, the

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\* In these calculations,  $n^*$  encompassed LOS 20, where retirement benefits are currently vested. A 10 percent yearly discount rate was used in these calculations, which may explain some of the "large" effect. Using a higher discount rate would serve to reduce the size of the predicted effect.

retention function is not derived from choice theory; it is an ad hoc specification. This gets back to the point that the model has no "error term" in it. It does not explain why some people choose to stay and others choose to leave after term  $t$ . Subsequent models attempt to correct these flaws.

#### THE ACOL MODEL

In the PVCOL model, the time horizon solved for by the model (i.e., the horizon of future service over which  $C_{t,n}$  is maximized) is implicitly the time horizon of a "taste neutral" individual, i.e., someone who is indifferent between military and civilian life. In reality, military personnel differ considerably in their attitudes or tastes for military service, and these differences should be accounted for in our analysis. The ACOL model uses the PVCOL model as a point of departure by introducing a taste for service factor explicitly into the model. The model then derives the time horizon that is relevant for retention decision-making and the military-civilian pay differential over that horizon, ACOL. The retention rate at LOS  $t$  is the proportion of individuals for whom the actual pay differential, or ACOL, is less than the required pay differential (as measured by the negative of each individual's taste for military service).

To introduce a taste factor into the model, we define  $\gamma_i$  as the  $i$ th individual's taste for service. We initially assume that  $\gamma_i$  is a "permanent" or "fixed" taste factor associated with the  $i$ th individual and does not depend upon current or prospective future term of service. This taste factor represents the negative of the military-civilian pay differential required during each term of service to make the  $i$ th individual indifferent between military and civilian life. Considering the adverse working conditions in some military occupations,  $\gamma$  is likely to be negative for many individuals. People with negative values of  $\gamma$  must be compensated with higher military than civilian pay to make them willing to remain in the service. Yet,  $\gamma$  may be highly positive for some. There are people who would remain in service in spite of a negative military-civilian pay differential.

Consider the recursive formulation of the PVCOL model. We incorporate the taste factor into the model by adding it to the return to

staying equation:

$$S_{it} = Y_i + M_{t+1} + \beta V_{t+1}(Y_i) \quad (3)$$

$$L_{it} = W_t + R_t$$

$$V_{it} = \text{MAX}(S_{it} ; L_{it})$$

Assume that the  $i$ th individual stays if his own personal cost of leaving,  $C_{it}(Y_i) = S_{it} - L_{it}$ , is positive. The retention rate will be the proportion of individuals for whom  $C_{it}(Y_i) > 0$ . Thus, defining  $\hat{Y}_i$  is the  $Y_i$  such that  $C_{it}(\hat{Y}_i) = 0$  and  $f_t(Y_i)$  as the distribution of  $Y$  among those at term  $t$ , the retention rate is

$$r_t = \int_{\hat{Y}_i}^{\infty} f_t(Y) dY \quad (4)$$

That is, the retention rate is just the proportion of individuals for

whom  $Y_i > \hat{Y}_i$ .

Given the future military and civilian pay streams, the  $\hat{Y}$  that separates stayers from leavers can be derived by iterating over various possible values of  $Y$  until finding the value for which  $C_{it}(Y_i)$  equals zero, that is,  $\hat{Y}$ .

The ACOL methodology provides an alternative derivation of this value. This methodology is essentially a closed form solution for the  $Y$  that yields a leaving cost of zero. Recall from the forward formulation of the ACOL model that the individual has  $T-t$  possible future leaving points. That is, he has  $T-t$  possible future time horizons to consider. We assume that the individual will remain in service only if there is at least one possible future horizon over which his cost of leaving is positive. The goal of the ACOL methodology is to find the values of  $Y$  such that there are no future horizons over which the cost of leaving is positive. The largest of these values will be the value of the taste factor that makes  $C_{it}(Y_i)$  equal to zero via the recursive procedure described above.

Recall that the cost of leaving at LOS  $t$  rather than LOS  $n$ ,  $C_{t,n}$ , can be written as



$$\sum_{j=t+1}^n \beta^{j-t-1} M_j + \beta^{n-t} [W_n + R_n] - W_t - R_t .$$

The decision to stay may be rewritten to say the individual stays if

there is at least one horizon over which  $-\left(\sum_{j=t+1}^n \gamma_1 \beta^{j-t-1}\right) < C_{t,n}$ .

This condition says that the individual will stay if there exists at least one horizon of future service over which the negative of the present value of his taste factor  $\gamma_1$  is less than the cost of leaving. This condition may be rewritten to say the individual stays if there is

at least one horizon over which  $-\gamma_1 < C_{t,n} \div \sum_{j=t+1}^n \beta^{j-t-1}$ . The value

$C_{t,n} \div \sum_{j=t+1}^n \beta^{j-t-1}$  is the annualized cost of leaving (ACOL). It is the annuity equivalent of  $C_{t,n}$ . We will call this  $A_{t,n}$ .

Now we assume that an individual will stay only if there exists at least one future time horizon over which  $\gamma_1 > -A_{t,n}$  or  $-\gamma_1 < A_{t,n}$ . That is, the individual examines each possible value of  $A_{t,n}$  and stays if and only if one of them exceeds  $-\gamma_1$ . Obviously, if any of them does, the maximum value does. Therefore, the  $\gamma_1$  that separates the stayers from the leavers is the maximum ACOL value. Call this value  $A_t$ . It is easy to show that  $C_{t,t}(-A_t)=0$ . Values of  $\gamma$  below  $-A_t$  imply negative leaving costs; values of  $\gamma$  above  $-A_t$  imply positive leaving costs. The retention rate is simply the proportion of individuals for whom  $\gamma$  exceeds  $-A_t$ :

$$r_t = \int_{-A_t}^{\infty} f_t(\gamma) d\gamma . \quad (5)$$

To repeat, the ACOL methodology is simply a closed form solution for the  $\gamma$  such that  $C_t(\gamma)=0$ .

If  $\gamma$  has the logistic distribution function, we can write that,

$$r_t = \frac{1}{1 + e^{-(a_0 + a_1 A_t)}} \quad (6)$$

Using this functional form, we may derive the effects of various pay changes. Again, pay changes that occur past the time horizon of the marginal individual (the individual for whom  $\gamma = \hat{\gamma}$ ) will not be

predicted to have any effect on  $r_t$ , unless, of course, the time horizon is altered by the pay change. This feature of the model is similar to the PVCOL model, except that if  $\hat{\gamma}$  is negative, the optimal time horizon will be shorter (remember that implicitly  $\gamma=0$  in the PVCOL model).

In other cases, we may derive the effects of various pay changes as follows. Suppose that  $n^*-t$  is the time horizon of the marginal individual, where  $n^*$  is derived via the ACOL methodology. A pay change in

the next term of service changes  $A_t$  by  $\frac{\partial A_t}{\partial M_{t+1}} = \frac{1}{\sum_{j=t+1}^{n^*} \beta^{j-t-1}}$ . Hence,

the retention rate is changed by  $\frac{\partial r_t}{\partial M_{t+1}} = a_1 r_t (1-r_t) \div \sum_{j=t+1}^{n^*} \beta^{j-t-1}$ . A

change in pay in the 2nd future term changes  $A_t$  by  $\frac{\partial A_t}{\partial M_{t+2}} = \frac{\beta}{\sum_{j=t+1}^{n^*} \beta^{j-t-1}}$

and hence  $r_t$  by  $\frac{\partial r_t}{\partial M_{t+2}} = a_1 r_t (1-r_t) \frac{\beta}{\sum_{j=t+1}^{n^*} \beta^{j-t-1}}$ . In general, a change

in pay in future term  $s < n^*$  changes  $r_t$  by  $\frac{\partial r_t}{\partial M_{t+s}} = \frac{a_1 r_t (1-r_t) \beta^{s-1}}{\sum_{j=t+1}^{n^*} \beta^{j-t-1}}$ . We

may compare the predicted effect from this model with the predicted effect for the PVCOL model. Remember in the PVCOL model that

$\frac{\partial r_t}{\partial M_{t+s}} = b_1 r_t (1-r_t) \beta^{s-1}$ . Therefore,  $\frac{\partial r_t}{\partial M_{t+s}}$  as derived from the PVCOL

model will exceed  $\frac{\partial r_t}{\partial M_{t+s}}$  as derived from the ACOL model as

$b_1 > a_1 \div \sum_{j=t+1}^{n^*} \beta^{j-t-1}$  or as  $b_1 \cdot \sum_{j=t+1}^{n^*} \beta^{j-t-1} > a_1$ .

To calculate the effect of a given dollar change in the whole pay table, ( $dM_{t+1} = \dots = dM_T = d\bar{M}$ ), we simply sum  $\frac{\partial r_t}{\partial M_{t+s}}$  from  $s=1$  to  $s=n^*-t$  as follows:

$$\begin{aligned} \frac{\partial r_t}{\partial \bar{M}} &= \frac{\partial r_t}{\partial A_t} \frac{\partial A_t}{\partial M_{t+1}} + \frac{\partial r_t}{\partial A_t} \frac{\partial A_t}{\partial M_{t+2}} + \dots + \frac{\partial r_t}{\partial A_t} \frac{\partial A_t}{\partial M_{n^*}} \\ &= a_1 r_1 (1-r_t) \frac{1 + \beta + \beta^2 + \dots + \beta^{n^*-t-1}}{\sum_{j=t+1}^{n^*} \beta^{j-t-1}} = a_1 r_t (1-r_t) . \end{aligned}$$

From this result, we see some of the characteristics of the ACOI model. Consider the case of a one-term time horizon ( $n^*=t+1$ ). In this case, an increase in pay yields the retention response

$$\frac{\partial r_t}{\partial M_{t+1}} = a_1 r_1 (1-r_1) . \text{ However, in the case of a two-term time horizon}$$

$$(n^*=t+2), \frac{\partial r_t}{\partial M_{t+1}} = \frac{a_1 r_1 (1-r_1)}{1+\beta} . \text{ Thus, if the time horizon of the}$$

Original individual is two terms, a pay change in the first term is predicted to yield a smaller response than if the horizon is only one term. This is because the pay change in the first term is annualized over two terms, not just one. The estimated retention response to a current term pay change thus depends on the marginal individual's time horizon. That time horizon and, consequently, the predicted response to pay change depend crucially on assumptions about the discount rate and future civilian opportunities. The higher the discount rate or the more rapidly civilian opportunities decline (or the slower they grow) with additional military service, the nearer  $n^*$  will be to the current term  $t$ . In addition, pay inversions due to high pay in term  $t+1$  but lower pay later (due say to higher first-term than second-term bonuses) will clearly shorten the time horizon.

Now the above specification of the retention function is unsatisfactory in two respects. First consider what would happen if the first-term taste distribution is normal or logistic and really is a "permanent" or "fixed" taste factor. If  $f_1(\gamma)$  is normal, then  $f_2(\gamma)$  will be a truncated normal, where the truncation point is  $-A_1$ . If  $\gamma$  really were a permanent taste factor, the distribution of  $\gamma$  would collapse as  $t$  increases. Indeed, as long as  $A_t$  rises monotonically with  $t$ , the

retention rate should be unity between the first-term point and the 20th year of service. This is not a very satisfactory implication, especially in light of the fact that retention rates do not currently approach unity until around the 15th year of service. We may show that the first- and second-term retention rates are,

$$r_1 = P(\gamma_1 > -A_1) = \int_{-A_1}^{\infty} f_1(\gamma_1) d\gamma_1$$

$$r_2 = \frac{P(\gamma_2 > -A_2, \gamma_1 > -A_1)}{P(\gamma_1 > -A_1)} = \frac{\int_{-A_2}^{\infty} \int_{-A_1}^{\infty} N(\gamma_1, \gamma_2) d\gamma_1 d\gamma_2}{P(\gamma_1 > -A_1)}$$

$$= \frac{\int_{-A_2}^{\infty} \int_{-A_1}^{\infty} \frac{e^{-\frac{1}{2} \frac{1}{(1-\rho)^2} \left[ \left( \frac{\gamma_1 - \mu_1}{\sigma_1} \right)^2 + \left( \frac{\gamma_2 - \mu_2}{\sigma_2} \right)^2 - 2\rho \left( \frac{\gamma_1 - \mu_1}{\sigma_1} \right) \left( \frac{\gamma_2 - \mu_2}{\sigma_2} \right) \right]}}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} d\gamma_1 d\gamma_2}{P(\gamma_1 > -A_1)}$$

The conditional retention probability  $r_2$  is the probability of staying for both terms, given  $r_1$ . The density of  $f_1(\gamma)$  is normal, but the conditional density of  $\gamma_2$  is not.\*

If the correlation  $\rho$  between  $\gamma_1$  and  $\gamma_2$  is zero, then  $r_1$  and  $r_2$  will be independent. If  $\rho=0$ , then there is no link between  $A_1$  and  $r_2$ . If  $\rho$  is positive, then  $r_2$  will decline as  $A_1$  rises. To see this intuitively, suppose  $\gamma_1$  and  $\gamma_2$  are positively correlated. Higher second-term pay raises  $A_1$  and this pay raise serves to retain more personnel with lower values of  $\gamma_1$ . Since  $\gamma_1$  and  $\gamma_2$  are positively correlated, the cohort facing the second-term retention decision will contain personnel who on average have lower tastes for service. Given  $A_2$ , this cohort will thus have a lower retention rate.

This specification of the retention function introduces some econometric problems. First, for more than two periods the retention equations become messy indeed. Second, even with just two terms, there are 5 parameters to estimate  $\mu_1$ ,  $\mu_2$ ,  $\sigma_1$ ,  $\sigma_2$  and  $\rho$ , and the estimation pro-

\* The density function of  $\gamma_2$  is normal for a fixed value of  $\gamma_1$ , but it is not normal over a range of values of  $\gamma_1$ .

cedure is complicated. Third, estimation of the parameters requires longitudinal data, a feature as well of the SCOL model to be discussed below.

While not as satisfying as full estimation of the bivariate normal model specified above, the approach which I took in reference 6 was to simply estimate the first- and second-term retention equations using probit analysis and include  $A_1$  as an additional variable in the second-term equation. While this methodology is not as efficient as joint estimation of the  $r_1$  and  $r_2$  functions using panel data, it is defensible on the grounds that panel data were unavailable. In almost all cases in reference 6, the coefficient on the  $A_1$  variable was negative, indicating that  $r_2$  does indeed fall when higher second-term pay induces more first-term retention.

The ACOL model has been applied extensively to give predictions of the effects of alternative compensation systems (see, e.g., reference 3). To make such predictions, the parameter  $a_1$  of the logistic distribution was estimated using regression analysis. Then, retention rates ( $r_t$ ,  $t=1\dots T$ ) during a given fiscal year (e.g. FY 1979) are used as a set of "baseline" retention rates. These baseline rates are used to project the steady-state or future transitional force structure that would evolve without a pay change. Then, the ACOL model is exercised to predict the retention rates and force structure that would evolve with a pay change, and the resulting forces are compared. In the analysis, the baseline retention rates will be altered both because pay changes that occur in later terms will affect the baseline rates and because pay changes that occur prior to term  $t$  will alter the taste distribution among those surviving to term  $t$ .

Policy analysis with the bivariate normal specification of the retention function is unrealistic because in the analysis must consider pay changes at more than two terms of service. We could specify the retention function as a T-dimensional multivariate normal, but this specification would become mathematically intractable. Therefore, in practical applications we have done the following. We assume that the relationship between  $r_t$  and  $A_t$  is normal or logistic. Then, we derive the predicted retention rate  $r_t$  via the following two-step procedure. First, the baseline retention rate in a given YOS cell,  $r_t$ , is adjusted as follows to account for the effect of pay changes in prior terms on the taste distribution among those surviving to term  $t$ :

$$r'_t = r_t (1 + e_{r_t, r_{t-1}} \cdot \frac{\Delta r_{t-1}}{r_{t-1}})$$

where:

$r'_t$  = retention rate in term  $t$  "adjusted" for the influence of past pay

$r_t$  = the unadjusted or baseline retention rate

$$e_{r_t, r_{t-1}} = \frac{\% \Delta r_t}{\% \Delta r_{t-1}} = \frac{\Delta r_t}{\Delta r_{t-1}} \cdot \frac{r_{t-1}}{r_t}$$

Second, using a normal or logistic retention function, we predict the new retention rate  $r_t$  by determining how  $r'_t$  is affected by a change in  $A_t$ .

Let us examine the first step more carefully.  $e_{r_t, r_{t-1}}$  is the elasticity of  $r_t$  with respect to  $r_{t-1}$ . This elasticity ranges between 0

and -1 in value, and may be derived as follows. Let  $r_t \cdot r_{t-1}$  be the fraction of personnel surviving to term  $t-1$  who stay beyond term  $t$ . The percentage increase in this fraction due to a change in pay in term  $t-1$

is  $\frac{\Delta r_t}{r_t} + \frac{\Delta r_{t-1}}{r_{t-1}}$ . If the retention decisions in the two terms are

independent, then  $\frac{\Delta r_t}{r_t}$  equals 0. In this case,  $e_{r_t, r_{t-1}}$  equals 0 and

$r'_t$  equals  $r_t$ . At the other extreme, everyone who remains for term  $t$  because of a pay increase in term  $t$  leaves at the end of term  $t$ , then

$\frac{\Delta r_t}{r_t}$  declines by the percentage increase in  $r_{t-1}$ ,  $\Delta r_{t-1}/r_{t-1}$ , and the

survival fraction  $r_t \cdot r_{t-1}$  remains unchanged. In this case,  $e_{r_t, r_{t-1}}$

equals -1 and  $r'_t$  equals  $r_t (1 - \frac{\Delta r_{t-1}}{r_{t-1}})$ . In general, with empirical

estimates of  $e_{r_t, r_{t-1}}$  ranging between 0 and -1 in value,  $r_t$  can be

adjusted to reflect the influence of past pay. Estimates of this elasticity derived in reference 6 range from about -.2 to -.8 in value, with

a means value of  $-.5$ .<sup>\*</sup> These estimates imply that some, though not all, of the personnel induced to remain in service by higher pay in term  $t$  will leave at the end of term  $t$ .

We can relate this process of adjusting  $r_t$  to account for the influence of past pay to the bivariate normal retention function. A value of  $e_{r_t, r_{t-1}}$  of 0 corresponds to the case of a zero correlation

between tastes at term  $t$  and tastes at term  $t-1$  ( $\rho=0$ ). A value of  $e_{r_t, r_{t-1}}$  of  $-1$  corresponds to the case of  $\rho$  equal to 1. Elasticities between 0 and  $-1$  in value correspond to values of  $\rho$  between 0 and 1.

#### THE SCOL MODEL

The previous models may be criticized on two grounds. The first is that both treat the future military and civilian income streams as known with certainty, and hence that the time horizon is known with certainty. In fact, the future income streams may not be known with certainty. Second, random events other than shocks to the military or civilian income streams may occur that will induce individuals to leave after each term. Further, if individuals are aware of the probabilities with which they will leave after each future term, they will incorporate these probabilities into their cost of leaving calculations. Each individual's cost of leaving becomes a probabilistic or "stochastic" cost of leaving based on his perceived probability of leaving after each term of service. In this model, the cost of leaving becomes a weighted average of the leaving costs over various time horizons, where the weights represent the probabilities of remaining in service until the end of each possible future term and then leaving.

The second criticism was alluded to in the introduction. The base case pattern of retention rates from the first-term to term  $T$  is not predicted from the model (i.e., it is not endogenous), but merely taken as given. Thus, even with no increase in the cost of leaving, second-term retention rates may be twice the level of first-term retention

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<sup>\*</sup> These estimates have been criticized on the ground that they were not constrained to lie in the interval  $(-1,0)$ , and that they were not obtained via an efficient estimation procedure, e.g., joint estimation of a bivariate normal. Yet they yield what appear to be plausible estimates of the elasticity,  $e_{r_t, r_{t-1}}$ .

rates. Yet, the models provide no self-contained explanation for why there should be a tendency for retention rates to rise in the face of no change in the cost of leaving. A corollary of this criticism is that the previous models do not provide a rigorous link between pay in one term and retention in the next term, although the adjustment procedure discussed above in the context of the ACOL model may mitigate some of the force of this criticism. The SCOL model now to be discussed represents an improvement over previous models in that (1) the uncertainty of the future time horizon is accounted for, (2) a retention function is specified in which the pattern of retention rates by term of service is explained within the model (i.e., is endogenous) and (3) the link between pay in one term and retention in future terms is made rigorous. These benefits do not come without some cost. First, the model is mathematically more complicated than previous models. Second, for purposes of policy analysis, it would be virtually impossible to fit the parameters of the model such that it perfectly predicts the historical base case that one might want to start with (e.g., FY 1979 retention rates). The model's predictions for a new pay regime must be compared with the model's own base case retention rates, not with historical rates. The model is suited to steady-state analysis of a hypothetical force, not dynamic policy analysis of an actual force.

Following a paper by Heckman and Willis (reference 8), Gotz and McCall (reference 5) reformulated the model of the retention decision and thereby obtained a more complicated retention function. They begin by assuming that  $\gamma$  is a "permanent" or "fixed" taste parameter that is not affected by current or prospective future years of service. Then, they introduce the idea of a transitory disturbance. The transitory disturbance  $\varepsilon_t$  is a random one-term addition to the individual's return to staying in term  $t+1$ . For instance, the individual may draw a bad assignment in term  $t+1$  (or at the end of term  $t$ ), but this bad assignment does not affect his expectation about future assignments. Therefore, it is assumed that the individual "draws" one transitory error term per term ( $\varepsilon_t$ ) and that these errors are uncorrelated across terms ( $\text{cov}(\varepsilon_t, \varepsilon_{t'}) = 0, t \neq t'$ ).\*

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\* As evidenced by the pattern of Navy sea-shore rotation policies, for instance, this assumption is unrealistic. However, the model would become exceedingly complex without it. One question is how much prediction error is introduced by this assumption, a subject we return to below.



On this assumption, the retention decision is derived recursively as follows. Suppose we consider the  $i$ th individual at the end of term  $T-1$ . His cost of leaving is,

$$C_{iT-1}(\gamma_i, \epsilon_{iT-1}) = \epsilon_{iT-1} + \gamma_i + M_T + \beta L_T - L_{T-1}$$

$C_{iT-1}(\gamma_i, \epsilon_{iT-1})$  is the cost of leaving conditional on both  $\epsilon_{iT-1}$  and  $\gamma_i$ , while  $C_{iT-1}(\gamma_i)$  is the cost of leaving unconditional on  $\epsilon_{iT-1}$ . The decision will be to stay if  $C_{iT-1}(\gamma_i, \epsilon_{iT-1})$  is positive, or if  $\epsilon_{iT-1} > -C_{iT-1}(\gamma_i)$ . Therefore, if  $G(\epsilon_{iT-1})$  is the distribution function of  $\epsilon_{iT-1}$ , the probability that the  $i$ th individual will stay for term  $T$  is,

$$\alpha_{iT-1} = \int_{-C_{iT-1}(\gamma_i)}^{\infty} dG(\epsilon_{iT-1}) .$$

Now, in the original formulation of the PVCOL model, the individual's optimal return was simply the maximum of  $S_{iT-1}$  and  $L_{iT-1}$ . This is no longer true. Since the individual gets the return  $S_{iT-1} = \epsilon_{iT-1} + \gamma_i + M_T + L_T$  with probability  $\alpha_{iT-1}$  and the return  $L_{T-1}$  with probability  $1-\alpha_{iT-1}$ , his expected return from staying --  $E(V_{iT-1})$  -- is calculated as

$$\begin{aligned} E(V_{iT-1}) &= \int_{-C_{iT-1}(\gamma_i)}^{\infty} (\epsilon_{iT-1} + \gamma_i + M_T + L_T) dG(\epsilon_{iT-1}) \\ &\quad + \int_{-\infty}^{-C_{iT-1}(\gamma_i)} L_{T-1} dG(\epsilon_{iT-1}) \end{aligned}$$

This is equal to

$$\begin{aligned} E(V_{iT-1}) &= \int_{-C_{iT-1}(\gamma_i)}^{\infty} \epsilon_{iT-1} dG(\epsilon_{iT-1}) + \alpha_{iT-1} (\gamma_i + M_T + L_T) \\ &\quad + (1-\alpha_{iT-1}) L_{T-1} . \end{aligned}$$

The first term in this sum is the expected value of the transitory disturbance given that the individual stays. In this formulation, this expected value is always positive. If  $\epsilon_{iT-1}$  is distributed normally with mean 0 and standard deviation  $\sigma_\epsilon$ , we may show that this expected value is equal to  $\sigma_\epsilon g(Z_{iT-1})/\alpha_{iT-1}$  where  $Z_{iT-1} = C_{iT-1}(\gamma_i)/\sigma_\epsilon$ . That is,  $Z_{iT-1}$  is the "standardized" unconditional cost of leaving for individual  $i$ .

Like the PVCOL model, SCOL can be solved recursively to obtain costs of leaving in earlier terms. In general, the unconditional cost of leaving at term  $t$  may be written as

$$C_{it} = Y_i + M_{t+1} + \beta E(V_{t+1}) - L_t.$$

Again, the retention decision is to stay if  $C_{it}(Y_i) + \varepsilon_{it} > 0$ , or if  $\varepsilon_{it} > -C_{it}(Y_i)$ .

While the model is solved recursively for  $C_{iT-1}$ ,  $C_{iT-2}$ ,  $C_{iT-3}$ , etc., it is revealing to reformulate it as a forward-looking model. To do this, I will ignore for the moment the expected value of truncated error terms  $\sigma_\varepsilon g(Z_{it})$  to simplify the argument.\* For simplicity, I also omit the  $i$  subscript. Consider the possibilities facing an individual at the end of term  $t$ . If he stays one more term, he gets the income stream  $S_{t,t+1} = Y + M_{t+1} + \beta L_{t+1}$ . If he stays two more terms, he gets the income stream  $S_{t,t+2} = Y + M_{t+1} + \beta(Y + M_{t+2} + \beta L_{t+2})$ . If he stays  $n$  more terms, he gets the income stream

$$S_{t,n} = Y + M_{t+1} + \sum_{j=t+2}^n \beta^{j-t-1} (Y + M_j) + \beta^{n-t} L_n.$$

Assuming that the individual stays for term  $t+1$ , the probability that he will leave at the end of term  $t+1$  is  $(1-\alpha_{t+1})$ . Call this probability  $\pi_{t,t+1}$ . Given that he stays for term  $t+1$ , the probability that he will stay until the end of term  $t+2$  and then leave is  $\alpha_{t+1}(1-\alpha_{t+2})$ . Call this  $\pi_{t,t+2}$ . In general, let  $\pi_{t,n+1} = \alpha_{t+1} \cdot \alpha_{t+2} \cdot \dots \cdot \alpha_n(1-\alpha_{n+1})$  be the probability that the individual will stay from the end of term  $t$  to the end of term  $n$  and then leave. Note that  $\alpha_T$  equals zero. Therefore, the probability of an individual at term  $t$  remaining through term  $T$  is  $\pi_{t,T} = \alpha_{t+1} \alpha_{t+2} \cdot \dots \cdot \alpha_{T-1}$ . The expected present value of the individual's income stream, given that he stays for term  $t$ , is

$$E(S_t) = \sum_{n=t+1}^{T-1} \pi_{t,n} S_{t,n}.$$

This simply says that the expected return to staying at the end of term  $t$  is a weighted average of the returns from following all possible

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\* It is shown below that under certain assumptions this term will cancel out anyway.

future stay-leave sequences, where the weights are the probabilities of following the possible sequences.

The individual's cost of leaving may be written as

$$C_t(\gamma) = E(S_t) - L_t = \sum_{n=t+1}^{T-1} \pi_{t,n} S_{t,n} - L_t .$$

We may simplify this further. Since  $\sum_{n=t+1}^{T-1} \pi_{t,n} = 1$ , the cost of leaving is

$$C_t(\gamma) = \sum_{n=t+1}^{T-1} \pi_{t,n} (S_{t,n} - L_t) = \sum_{n=t+1}^{T-1} \pi_{t,n} C_{t,n} .$$

Thus, the cost of leaving at term  $t$  is simply a weighted average of the leaving costs over all possible future stay-leave paths or time horizons where, again, the weights represent the probabilities that the individual will follow these paths.

This "forward looking" representation of the model is useful because it permits us to examine how pay changes affect the unconditional cost of leaving. Later, we will derive their effects on retention rates. We cannot do this just yet since we have not yet specified the retention function. Consider the effect of a change in  $M_{t+1}$ . Clearly  $\partial C_t(\gamma) / \partial M_{t+1} = 1$ . That is, the unconditional cost of leaving changes dollar for dollar with a change in  $M_{t+1}$ . This result is identical to the PVCOL model and the ACOL model where the time horizon is one term. Thus, in the case where the ACOL model horizon is one term, all three models give (approximately) the same prediction of the effect of a pay change in the next term (depending, of course, upon how the retention function is specified).

Next, consider the effect of a pay change in the second-future term. We can show that

$$\frac{\partial C_t(\gamma)}{\partial M_{t+2}} = \beta \alpha_{t+1} + \frac{\partial \alpha_{t+1}}{\partial M_{t+2}} \cdot M_{t+2} .$$

This partial derivative simply says that the effect of a pay change in the next term is the discount rate  $\beta$  times the probability that the

individual will stay for the next period plus  $\beta$  times the effect of a change in  $M_{t+2}$  on  $\alpha_{t+1}$ . Now, the important point is that in this model a pay change in the next term clearly has a smaller effect on the cost of leaving than in the PVCOL model.\* (Recall that in that model the effect was  $\beta$ ). This result says that the individual weighs a pay change during the next term by the probability that he will leave as is the pay change. Hence (again depending on how the retention function is specified) this model will give a smaller estimate of the effect of a pay change in the next term than will the PVCOL model. It is not clear how the prediction will compare with the ACOL model prediction, because in that model the predicted retention effect was smaller as well.

Let  $P_{t,n} = 1 - \sum_{j=t+1}^{n-1} \pi_{t,j}$  be the probability that an individual

will remain in service for at least  $n-t$  more terms. We can show that the effect of a pay change in term  $n$  has the following effect on  $C_t(\gamma)$ :

$$\frac{\partial C_t(\gamma)}{\partial M_{t+n}} = \beta^{n-1} \left[ P_{t,n} + \frac{\partial P_{t,n}}{\partial M_{t+n}} M_{t+n} \right].$$

That is,  $C_t(\gamma)$  changes by  $\beta^{n-1}$  times the sum of (1) the probability of "seeing" the pay change in term  $t+n$  and (2) the effect of the change in  $M_{t+n}$  on the probability of surviving to term  $t+n$ . Of course, we expect  $\partial P_{t,n} / \partial M_{t+n}$  to be positive.

The effect of a change in the whole future pay stream is simply the sum of  $T-t$  partial derivatives:

$$\frac{\partial C_t(\gamma)}{\partial \bar{M}} = \sum_{n=1}^{T-t} \frac{\partial C_t(\gamma)}{\partial M_{t+n}} = \sum_{n=1}^{T-t} \beta^{n-1} \left[ P_{t,n} + \frac{\partial P_{t,n}}{\partial M_{t+n}} \cdot M_{t+n} \right].$$

Since the survival probabilities are less than unity, the effect of a change in the whole pay table should again be smaller than it was in the PVCOL model.\*\*

\* This assumes the time horizon of the "taste neutral" individual exceeds one period.

\*\* A qualifying factor is that the PVCOL model sums partial derivatives only from term  $t$  to term  $n^*$ , where  $n^*$  is the first future term for which  $RL_{t,n^*} > RS_{t,n^*}$ . This model sums partial derivatives from term  $t+1$  to the terminal term  $T$ . Therefore, this statement should not be taken too literally.

Now we derive the retention equation. Suppose we first consider the  $i$ th individual with taste factor  $\gamma$ . If this individual has the sequence of unconditional leaving costs  $C_1(\gamma), C_2(\gamma) \dots, C_t(\gamma)$  from term 1 to term  $t$ , the probability that he will remain in service from term 1 to term  $t$  is,

$$u_1 \cdot u_2 \cdot \dots \cdot u_t = \int_{-C_1(\gamma)}^{\infty} dG(\epsilon_1) \int_{-C_2(\gamma)}^{\infty} dG(\epsilon_2) \cdot \dots \cdot \int_{-C_t(\gamma)}^{\infty} dG(\epsilon_t) .$$

Now this is the probability that the given individual will stay for  $t$  terms. Consider next a whole cohort of individuals at the end of the first term. If  $f(\gamma)$  is the (fixed) distribution of tastes among the individuals in this cohort, the fraction that is expected to survive to term  $t$  is

$$s_t = \int_{-\infty}^{\infty} \left[ \int_{-C_1(\gamma)}^{\infty} dG(\epsilon_1) \int_{-C_2(\gamma)}^{\infty} dG(\epsilon_2) \cdot \dots \cdot \int_{-C_t(\gamma)}^{\infty} dG(\epsilon_t) \right] dF(\gamma) .$$

That is, we find  $s_t$  by weighting different individuals' survival probabilities to term  $t$  by their representation in the initial cohort. The retention rate at term  $t$  is the conditional probability of staying at term  $t$  given survival to term  $t-1$ ,  $r_t = s_t / s_{t-1}$ .

If we in fact knew the unconditional taste distribution at term  $t$ ,  $f_t(\gamma)$ , we could calculate  $r_t$  as,

$$r_t = \int_{-\infty}^{\infty} \int_{-C_t(\gamma)}^{\infty} dG(\epsilon_t) dF_t(\gamma) .$$

As discussed below, this conditional distribution is skewed to the right, and its skewness depends upon the cost of leaving in prior terms. In practice, this conditional distribution is so complicated mathematically that we will work with the unconditional survival probabilities and derive the retention rates by dividing  $s_t$  by  $s_{t-1}$ .

We can at least examine intuitively the relationship between retention rates at various terms of service. To do this, suppose that  $f(\gamma)$ , the initial taste distribution, is distributed  $N(\mu_\gamma, \sigma_\gamma)$  and that the transitory error term  $\epsilon_t$  is distributed  $N(0, \sigma_\epsilon)$ , each and every  $t$ . If  $\gamma + \epsilon_t$  is the "error term" in the retention decision at each term of service, we may show that the correlation between the errors at various

terms,  $\rho$  equals  $\frac{\sigma_Y^2}{\sigma_Y^2 + \sigma_\varepsilon^2}$ . Consider first the polar case where  $\rho = 0$ , i.e., everyone has the same taste factor. In this case, differences in tastes explain none of the individual variation in retention probabilities -- all of the variation is explained by random draws on  $g(\varepsilon_t)$ . Recall from earlier discussion that  $e_{r_t, r_{t-1}}$  equals the elasticity of the retention rate in term  $t$  with respect to the retention rate in term  $t-1$ . In this case, the elasticity equals zero. That is, the retention rates in different terms are independent of one another.

Consider the second polar case where  $\sigma_\varepsilon = 0$  and  $\sigma_Y > 0$ . Here, all of the individual variation in retention probabilities is due to variation in the permanent taste factor  $\gamma$ . Here,  $\rho$  equals 1. In this case, individuals stay only so long as  $C_t(\gamma)$  exceeds zero. Hence, a higher first-term bonus will make  $C_1(\gamma)$  positive for some individuals for whom  $C_1(\gamma)$  was previously negative. If  $C_2(\gamma)$  is negative for these individuals, all will leave at the end of the second-term. Hence, in this polar case,  $e_{r_2, r_1} = -1$ . As a general rule,  $e_{r_2, r_1}$  will lie between 0 and -1 in value; its value will approach -1 as  $\sigma_Y$  rises relative to  $\sigma_\varepsilon$ .

In the case where  $\sigma_\varepsilon$  exceeds zero, the original taste distribution  $f(\gamma)$  does not get truncated as term of service ( $t$ ) rises, only "thinned out" in its lower tail. This occurs because individuals with low (negative) values of  $\gamma$  have lower retention probabilities than individuals with higher (positive) values. The conditional taste distribution  $f_t(\gamma)$  becomes skewed to the right as  $t$  increases. Its mean rises and its standard deviation declines. The extent to which  $f_t(\gamma)$  is skewed depends upon compensation policies. Higher military pay serves to retain more people with low values of  $\gamma$  and hence reduces the skewness of  $f_t(\gamma)$ , with the consequent results that the conditional mean is lower and the conditional standard deviation is higher. Because the mean of the conditional distribution depends inversely on past compensation, higher past compensation necessarily leads to lower retention in term  $t$ . In this regard, this specification of the retention equation provides a "closed form" solution to the link between compensation policies in one term and retention rates in future terms.

We now examine the retention impact of pay changes. Given the mathematical complexity of the model, we can only state the results formally. The retention impact of a pay change, e.g.  $\partial r_t / \partial M_{t+1}$ , does not have the simple analytical solution that one finds in the earlier

models based on the logistic supply function. But, to begin with, suppose that  $f_t(\gamma)$  is the conditional taste distribution at term  $t$ . If  $\varepsilon_t$  is distributed normally, we may show that

$$\frac{\partial r_t}{\partial M_{t+1}} = \int_{-\infty}^{\infty} \frac{e}{\sqrt{2\pi}} - \frac{1}{2} \left( \frac{C_t(\gamma)}{\sigma_\varepsilon} \right)^2 \frac{1}{\sigma_\varepsilon} dF_t(\gamma) .$$

The first term in this integrand is the ordinate of the standard normal distribution evaluated at  $C_t(\gamma)/\sigma_\varepsilon$ , the standardized cost of leaving. It shows how a single individual's retention probability changes with a change in  $C_t(\gamma)/\sigma_\varepsilon$ . The second term,  $\frac{1}{\sigma_\varepsilon}$ , shows the rate at which  $C_t(\gamma)/\sigma_\varepsilon$  changes in  $M_t$ . (We have previously shown that  $\partial C_t(\gamma)/\partial M_{t+1} = 1$ .) Therefore, the product of these two terms shows how a single individual's retention probability is affected by a change in  $M_{t+1}$ . Summing over individuals (i.e. integrating over the range  $\gamma$ ) gives  $\partial r_t/\partial M_{t+1}$ .

By a similar process, we may also find the effect on  $r_t$  of a pay change in term  $t+n$  and the effect of a change in the whole future pay stream:

$$\frac{\partial r_t}{\partial M_{t+n}} = \int_{-\infty}^{\infty} \frac{e}{\sqrt{2\pi}} - \frac{1}{2} \left( \frac{C_t(\gamma)}{\sigma_\varepsilon} \right)^2 \left[ \frac{1}{\sigma_\varepsilon} \frac{\partial C_t(\gamma)}{\partial M_{t+n}} \right] dF_t(\gamma)$$

$$\frac{\partial r_t}{\partial M} = \int_{-\infty}^{\infty} \frac{e}{\sqrt{2\pi}} - \frac{1}{2} \left( \frac{C_t(\gamma)}{\sigma_\varepsilon} \right)^2 \left[ \frac{1}{\sigma_\varepsilon} \frac{\partial C_t(\gamma)}{\partial M} \right] dF_t(\gamma) .$$

The terms  $\frac{\partial C_t(\gamma)}{\partial M_{t+n}}$  and  $\frac{\partial C_t(\gamma)}{\partial M}$  were derived above. Due to their complexity, we do not repeat them here.

While not obvious, a result of this model is that the retention effect of a pay change will increase as term of service rises. That is,

$$\frac{\partial r_t}{\partial M_{t+1}} > \frac{\partial r_{t'}}{\partial M_{t'+1}} \text{ for } t > t' .$$

As noted above, as the initial taste

distribution becomes skewed, its standard deviation declines -- that is,  $s_t^q < s_q$ , where  $s_t^q$  is the standard deviation of  $f_t(q)$ . As a result of this lower variation in tastes, a given pay change will cause movement over a larger portion of the  $f_t(q)$  distribution at  $t$  than at  $t' < t$ . Other things equal, this implies a larger change in  $r_t$  than in  $r_{t'}$ . This leads, for instance, to the prediction that a second-term bonus will have a larger impact on the second-term retention rate than an equivalent bonus on the first-term retention rate. The reason for expecting this larger impact is separate and distinct from the fact that retention changes will get larger as one moves from the tail of a normal distribution (e.g., a 20 percent base retention rate) to the center of the distribution (e.g., a 50 percent base retention rate).

Let us turn now to certain assumptions about the specification of the SCOL model. In this model, the transient error disturbance  $e_t$  enters as a per term disturbance to the individual's military income stream. Yet, it seems reasonable to assume that random shocks could occur to the return to leaving equation as well as the return to staying equation. We could introduce a random civilian disturbance  $e_t^c$  to the model, but we do so only at the expense of greater complexity. There is one case, however, where adding a civilian disturbance greatly simplifies the model. Suppose we distinguish between the random civilian disturbance  $e_t^c$  and the random military disturbance  $e_t^m$ . Under the assumptions that they both have the same probability distribution and that they are uncorrelated, we may show that the expected value of truncated error terms cancel out in the expected optimal return equation.\*

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\*The expected value of  $e_t^m$  given that the individual stays is 
$$\frac{-s_m g_m \left( \frac{C_t(q)}{s_m} \right)}{1-a_m}$$

where  $s_m$  is the standard deviation of  $e_t^m$ . The expected value of  $e_t^c$

given that the individual leaves is 
$$\frac{-s_c g_c \left( \frac{C_t(q)}{s_c} \right)}{1-a_c}$$
. Since the individual gets the expected value of  $e_t^m$  with probability  $a_m$  and the expected value of  $e_t^c$  with probability  $1-a_c$ , it may be shown that these terms sum to zero when  $e_t^m$  and  $e_t^c$  have identical (non-correlated) distributions.



The assumption that there are random disturbances to both the civilian and military income streams is convenient because it simplifies the calculation of the unconditional cost of leaving (one can use the backward method or the forward method discussed above). Also, I see no way of ever jointly estimating the parameters of both distributions.

A second assumption up to this point has been that everyone has basically the same civilian opportunities (aside from random disturbances to the civilian income stream). In reality, individuals will differ in their civilian opportunities. Aside from the taste factor  $\gamma$ , differences in abilities are another persistent source of variation. The process that sorts out stayers from leavers may as much be a sorting due to differences in civilian opportunities as differences in tastes. Empirical evidence for this view is found in reference 7. The probability of reenlisting is found to vary systematically by mental group, education level, and race, and the effects of these variables are all in the expected direction.

The fact that there may be two sources of persistent variation in the model introduces problems of estimation and interpretation. First, one could simply assume that the term  $\gamma$  represents the sum of a pure taste factor and the deviation of the individual's true expected civilian earnings from the average for the cohort. In this case,  $\sigma_\gamma$  is the square root of the sum of the standard deviations of these two factors and the negative of twice their covariance. If we assume this and get an estimate of  $\sigma_\gamma$ , we find that the estimate of  $\sigma_\gamma$  overstates the standard deviation of the pure taste distribution. The basic problem here is that when the model is simulated to determine the effects of pay changes, the resulting pay elasticities will be too small -- pay elasticities vary inversely with  $\sigma_\gamma$ .

In empirical work, the upward bias in estimation of  $\sigma_\gamma$  can be reduced by getting an estimate of civilian opportunities for each individual (each group of identically attributed individuals) in the sample. This was done, for instance, in reference 6. Some of the persistent variation in civilian opportunities will be controlled for, but individual errors still remain, leading again to upward bias in estimation of  $\sigma_\gamma$ .

#### Air Force - CBO Models

On the surface, the stochastic formulation of the cost of leaving is similar to retention models constructed by the Air Force and the Congressional Budget Office (CBO). Both the Air Force and CBO models

compute the returns to staying by using estimated probabilities of going to each future term and then leaving. To do this, the models begin at term T and work recursively to predict new retention rates and use the predicted retention rates to compute the  $\alpha$ 's used in the cost of leaving equation in each preceeding term. Thus, the models compute  $C_T$  and predict  $r_T$  using a logistic supply equation.  $r_T$  becomes the  $\alpha_{T-1}$  used in the calculation of  $C_{T-1}$ . Next,  $r_{T-1}$  is predicted using  $C_{T-1}$ . The  $\alpha$ 's required for the calculation of  $C_{T-2}$  are calculated using  $r_{T-1}$  and  $r_T$ . In the same fashion,  $r_{T-2}$  is then predicted, and so forth. The models thus calculate expected costs of leaving for the "average individual," i.e., one whose future retention probabilities are equal to the cohort's average.

This methodology is computationally much simpler than the SCOL model procedure, but it has a shortcoming. The predicted retention rates used to estimate the  $\alpha$ 's represent the average  $\alpha$ 's only of those who choose to stay at term t. That is, they are conditional probabilities. They do not represent the  $\alpha$ 's of those who in fact choose to leave at term t. Since those who stay have, on the average, higher  $\alpha$ 's than those who choose to leave, this methodology probably leads to overstatement of the effect of far future pay changes on  $C_t$  and hence to overprediction of the effects of such pay changes on  $r_t$ . Yet, because the cost of leaving is a stochastic cost of leaving based on future retention probabilities, this model will yield a smaller predicted retention response to a future pay change than the PVCOL model. Just how the Air Force - CBO model predictions compare with ACOL or SCOL model predictions for structural changes such as retirement overhaul is not clear at the moment.

## SIMULATIONS OF THE SCOL MODEL\*

We begin the simulation analysis of the SCOL model by assuming that  $\gamma$  is initially distributed  $N(\mu_\gamma, \sigma_\gamma)$ , and that  $\epsilon_t$  is distributed  $N(0, \sigma_\epsilon)$ . The model was simulated for various values of the parameters  $\mu_\gamma$ ,  $\sigma_\gamma$ , and  $\sigma_\epsilon$ . For brevity, we report the results of only one of the simulations. Retention rates obtained with other parameters differ somewhat, but the general retention patterns are similar. Importantly, for a feasible range of the parameters, the pay elasticities implied by the model are very similar.

This simulation analysis was performed using FY 1979 all-Navy data on promotion probabilities and bonus multiples. For FY 1979, we computed the "all-Navy" Zone A bonus multiple to be 1.53, and the Zone B multiple to be 1.27.\*\* For purposes of simulation, we divided the career into 14 decision points -- first-term (LOS 3-6), second-term (LOS 7-10), third-term (LOS 11-14), fourth-term (15-19), and terms 5-14 (LOS 20-29). Thus, we assume that at the end of each year between completion of LOS 20 and completion of LOS 29, the individual is eligible to leave the Navy. In practice,  $C_t(\gamma, \epsilon)$  is calculated at the midpoint of each of the first four terms. Thus the first-term cost of leaving is calculated at the beginning of LOS 5, the second-term cost of leaving at the beginning of LOS 9, etc. The reason for structuring the analysis this way, rather than calculating a cost of leaving for each year, is that individuals typically must make multi-year decisions prior to LOS 20, and, more pragmatically, the computations would become exceedingly cumbersome otherwise.

Table 1 shows FY 1979 all-Navy retention rates for the first five terms of service (where "term" is defined by the LOS intervals above), and the "base case" retention pattern for the parameters  $\mu_\gamma = -\$2800$ ,  $\sigma_\gamma = \$3500$  and  $\sigma_\epsilon = \$4500$ . These parameters were selected because they are the ones that gave the closest fit to the FY 1979 retention

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\* The author wishes to thank Dr. Philip M. Lurie of CNA who conceived the basic algorithm for the simulations reported on below.

\*\* These average multiples were obtained by weighting each rating's multiple by the number of reenlistment eligibles in FY 1979, summing, and dividing by the total number of eligibles in FY 1979. These multiples are slightly downward-biased due to the fact that we were unable to breakdown the data in certain ratings where reenlistees holding certain NECs receive larger bonuses than other reenlistees.

pattern. Table 1 also shows survival rates calculated from FY 1979 retention rates and survival rates derived from the "base case" retention rates. The survival rates are obtained by simply multiplying the relevant retention rates. In both cases, the survival rates show how many individuals would remain from term 1 to term t were the system in steady-state. The survival pattern produced by multiplying together the FY 1979 retention rates in table 1 is similar to the pattern that would be obtained by multiplying together yearly continuation rates. The simulations use a 10 percent personal discount rate throughout, and civilian opportunities are approximated by an age-earnings function fit to data on high school graduates.\*

The model simulated with the parameters specified does not perfectly predict the current retention pattern. Perhaps further experimentation with the model parameters will yield a better fit. In any event the purpose of this simulation is not to estimate the parameters empirically. Rather, given some chosen parameters, it is to show the retention predictions the model would make for various changes in the compensation system. We therefore simulated the model for:

- a 10 percent increase in second-term pay (LOS 5-8)
- a 10 percent increase in third-term pay (LOS 9-12)
- a 10 percent increase in pay in LOS cells 9-30
- a 10 percent increase in the whole pay table
- a two-tier retirement plan (the OSD retirement plan without early withdrawal privileges)
- the OSD retirement plan

Let us examine the results.

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\* We fit a regression of the form  $\log E_t = b_0 + b_1 t - b_2 t^2$  to data on earnings of high school graduates where  $\log E_t$  is the natural logarithm of earnings and t is years of labor force experience (age - 18). We fit such a function rather than using actual values because it was easier to use such a function rather than actual values in the simulation analysis. The fitted function explains 96 percent of the actual variation in age-earnings profiles of high school graduates.

TABLE 1

FY 1979 ALL-NAVY ENLISTED RETENTION AND  
SURVIVAL RATES BY TERM AND RATES PREDICTED BY SCOL  
MODEL FOR PARAMETERS  $\mu_Y = -2800$ ,  $\sigma_Y = 3500$ , AND  $\sigma_E = 4500$

Term	Actual retention rate <sup>a</sup>	Survival rate based on FY 1979 retention rates	Base case	
			Predicted retention rate	Predicted survival rate
1	.241	.241	.227	.227
2	.510	.123	.514	.117
3	.806	.099	.934	.109
4	.939	.093	1.00	.109
5	.331	.031	.422	.046

<sup>a</sup>Data provided by DMDC. Actual retention rates in this table are the proportions of people in LOS interval who had less than 13 months to go on their enlistment contracts at the start of the fiscal year who remained in the Navy at the end of the year. Note that the model predicts a unitary retention rate in the fourth term, yet the fourth-term rate is only 93.9 percent. This discrepancy is due to the fact that the model assumes no retirement eligibility prior to completion of 20 years of service, whereas there are individuals who have earned "constructive credit" time who are currently retiring prior to completion of 20 years.

Table 2 shows the predicted effect of a 10 percent rise in second-term pay, and compares this with the base case, which is reproduced from table 1. The model predicts a first-term retention rate increase from 22.7 percent to 28.2 percent. The implied pay elasticity ( $\% \Delta r_1 / \% \Delta M_2$ ) is 2.42. That is, the 10 percent rise in second-term pay is predicted to get a 24.2 percent rise in first-term retention. This is very similar to the first-term pay elasticities obtained with quite different models. The theoretical reasons for expecting this result were explained in the first section.

TABLE 2

## PREDICTED EFFECT OF 10 PERCENT RISE IN SECOND-TERM PAY

Term	Base case		10 percent second-term pay increase	
	$r_t$	$S_t$	$r_t$	$S_t$
1	.227	.227	.282	.282
2	.514	.117	.430	.121
3	.934	.109	.919	.112
4	1.00	.109	1.00	.112
5	.422	.046	.417	.047

A second result is that the second-term retention rate drops by 16.3 percent. This result is consistent with theory and is a necessary outcome of the construction of the model. From these results, we may show that the second-term reenlistment rate of the pay-induced first-term reenlistees is only 8.8 percent. The elasticity of the second-term retention rate with respect to the first-term rate ( $\% \Delta r_2 \div \% \Delta r_1$ ) may be calculated as  $(8.8 - .514) / .514 = -.824$ . It was shown in the first section that this elasticity can range between 0 and -1 in value, depending upon the relative values of the parameters  $\sigma_\gamma$  and  $\sigma_\epsilon$ . The elasticity obtained here is somewhat higher than ones obtained from empirical analysis with the ACOL model (reference 6). Note that the higher second-term pay will impact upon the third-term retention rate as well, although the predicted effect is rather small.

A general implication of the model is that the higher pay basically serves to retain personnel only during the interval over which pay is raised. The unconditional survival probabilities to term beyond the second term are not very different from the base case; most of the additional personnel retained by the higher second-term pay leave thereafter.

We turn now to the effect of a 10 percent rise in third-term pay. The model's prediction for this pay increase is shown in table 3. In table 3 we distinguish between the "short-run" effect of the third-term pay change and the "steady-state" effect. The "short-run" effect shows the immediate impact on the second-term retention rate of a 10 percent

increase in third-term pay. That is, it shows the effect on  $r_2$  of holding constant the pool of second-term eligibles. In the long-run (steady-state), however, a 10 percent increase in third-term pay can have the effect of raising the first-term retention rate  $r_1$ . If this happens, the pool of second-term eligibles will change, and the steady-state value of  $r_2$  after the 10 percent increase in third-term pay will be lower than the short-run value.

TABLE 3  
EFFECT OF A 10 PERCENT INCREASE IN THIRD-TERM PAY

Term	Base case		Short-run effect		Steady-state effect	
	$r_t$	$s_t$	$r_t$	$s_t$	$r_t$	$s_t$
1	.227	.227	.227	.227	.249	.249
2	.514	.117	.666	.151	.632	.158
3	.934	.109	.875	.132	.857	.135
4	1.00	.109	1.000	.132	1.00	.135
5	.422	.046	.388	.052	.384	.052

Examining the results in table 3, in the short-run, holding the pool of second-term eligibles the same as in the base case,\* the second-term retention rate rises from .514 to .666. In the steady-state,  $r_2$  is predicted to rise from .514 to .632. The pay elasticities implied by this 10 percent increase in third-term pay are very large. The short-run pay elasticity ( $\% \Delta r_2 / \% \Delta m_3$ ) is 2.96, while the steady-state elasticity is 2.29. These high elasticities are a result of the construction of the model. As noted earlier, the conditional distribution of  $y$  gets "thinned out" after the first-term decision point; the conditional mean rises relative to  $\mu_y$  and the conditional variance is smaller. As a result of the smaller variance of the conditional taste distribution

\* Computationally, we hold  $r_1$  and hence the pool of second-term eligibles constant by using the base case values of the cost of leaving at the first-term. I want to thank Glenn Gotz for suggesting this methodology.

(the taste distribution is now "tighter"), given changes in pay will lead to larger changes in retention than they did at the first term.

These prediction are to be contrasted with the second-term pay elasticities obtained from recent empirical analysis using the ACOL model (reference 7). This analysis obtained second-term pay elasticities that were only about half the value of those obtained in these simulations.

We turn in table 4 to the effect of a 10 percent increase in the whole pay stream beyond the second-term. Again, we may distinguish between the short-run and steady-state effects of the pay change. The short-run effect of the pay raise is to increase the second-term retention rate from .514 to .731, a large effect indeed. The implied pay elasticity is 4.22. This large a pay elasticity is due to the conditioning on the initial taste distribution. Again, the steady-state effects are somewhat smaller. Note that due to the increase in the whole post-second-term pay table, the retention pattern is uniformly higher than the base case retention pattern. This is to be contrasted with the previous case, where pay was raised only in the third term. There, post-second-term retention rates are below the base case rates. These results are, of course, to be expected.

TABLE 4  
EFFECT OF A 10 PERCENT INCREASE IN PAY IN LOS CELLS 9-30

<u>Term</u>	<u>Base case</u>		<u>Short-run effect</u>		<u>Steady-state effect</u>	
	<u>r<sub>t</sub></u>	<u>S<sub>t</sub></u>	<u>r<sub>t</sub></u>	<u>S<sub>t</sub></u>	<u>r<sub>t</sub></u>	<u>S<sub>t</sub></u>
1	.227	.227	.227	.227	.239	.239
2	.514	.117	.731	.166	.701	.167
3	.934	.109	.985	.163	.982	.164
4	1.00	.109	1.00	.163	1.00	.164
5	.422	.046	.462	.075	.460	.076



Now we turn to the effect of a 10 percent increase in the whole military pay table. The results are displayed in table 5. The first-term retention rate is predicted to rise from .227 to .300, a 32.2 percent increase. Thus, the first-term pay elasticity with respect to a 10 percent increase in the whole pay stream is 3.22. This is larger than the effect of a 10 percent increase in second-term pay, 2.42, but it is not too much larger. The whole pay stream elasticity seems much more plausible than that obtained with the original PVCOL model, but it is somewhat larger than that obtained with the ACOL model.

TABLE 5  
EFFECT OF A 10 PERCENT INCREASE IN WHOLE PAY TABLE

Term	Base case		Steady-state effect	
	$r_t$	$s_t$	$r_t$	$s_t$
1	.227	.227	.300	.300
2	.514	.117	.603	.181
3	.934	.109	.972	.176
4	1.00	.109	1.000	.176
5	.422	.046	.447	.079

Note that the second-term rate is 17.3 percent higher than the base case rate. Again, the steady-state effect of the 10 percent increase in the whole future pay stream is smaller than the short-run effect shown in table 4. It is due to the fact that many of the additional first-term reenlistees leave after the second-term.

We now turn to retention predictions for alternative retirement systems. We estimated the effects of two plans, the OSD plan and a similar plan without early withdrawal privileges. These plans are described in detail in reference 4. The results are displayed in table 6. The two-tier plan, which cuts retirement benefits from YOS 20 to age 60, generates less retention than the current system. The results indicate not much change in first- or second-term retention, but a significant drop in third-term retention. These results are reasonably close to the ACOL model results found in reference 3. The ACOL model predicts somewhat larger drops in first- and second-term retention, but a smaller drop in third-term retention. The predicted changes in the

cumulative survival probability to YOS 20 are about the same. As in the ACOL model, this model predicts a sharp increase in retention at YOS 20 (term 5 in table 6).

TABLE 6  
BASE CASE RETENTION RATES AND STEADY STATE RATES  
PREDICTED FOR TWO ALTERNATIVE RETIREMENT SYSTEMS

Term	Base case		Two-tier plan <sup>a</sup>		OSD plan	
	$r_t$	$S_t$	$r_t$	$S_t$	$r_t$	$S_t$
1	.227	.227	.226	.226	.306	.306
2	.514	.117	.497	.112	.652	.199
3	.934	.109	.838	.094	.683	.136
4	1.00	.109	1.000	.094	.998	.136
5	.422	.046	.574	.054	.597	.081

<sup>a</sup>Two-tier plan is identical to OSD plan except that no early withdrawals are allowed.

The SCOL model predicts a substantial increase in first- and second-term retention under the OSD retirement plan, but a sizeable drop in third-term retention. The predicted increase in early retention is much larger than that predicted by the ACOL model, but the predicted drop in third-term retention is larger as well. Of course, in the SCOL model (as well as the ACOL model), the size of the predicted drop in third-term retention will be directly related to the size of the predicted increase in first- and second-term retention. Overall, judging by the cumulative survival probability to YOS 20, the SCOL model predictions are somewhat more optimistic than the ACOL model predictions. The ACOL model predicted about the same cumulative survival probability to YOS 20 as under the current retirement system. However, because the manner in which cumulative survival probabilities were computed in the ACOL model was so different from the manner in which they are computed here, perhaps not too much should be made of the difference in the predicted survival probabilities. That the models do

give the same directional predictions is perhaps the important point to stress.\*

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\* We have not attempted a transition analysis of the effect of these retirement plans. It is clear, however, that during a transition period, retention would be higher than it would be under either retirement plan alone. This is because everyone currently in the force would be grandfathered at the time of implementation of the OSD plan, and because those who would prefer the OSD plan would be allowed to switch to it.

## CONCLUSIONS

The first section of the Research Contribution does not admit to conclusions in the usual sense, since it is a description and comparison of different analytic models. However, several points should be emphasized. The first is that the later models discussed here (ACOL and SCOL) are more descriptively accurate than the earlier ones. The PVCOL model examines nothing more than the monetary costs of staying in and leaving military service. The ACOL and SCOL models include a taste for military service factor, and the SCOL model adds a transitory disturbance to individuals' present value calculus to account for temporary effects on the stay-leave decision. This latter addition also means that the calculation of the future costs of staying or leaving becomes probabilistic, rather than certain; a condition which may more accurately describe behavior.

A second point is that the models become more internally consistent and complete. For example, the SCOL model makes the link between pay in one term and retention in future terms a part of the model, rather than an ad hoc procedure. Also, retention by term of service is endogenous to the model, not exogenous as it was previously.

Finally, (and this point leads into the simulation of the SCOL model) the later models provide more sensible predictions than earlier models. Rather than repeating the findings from the simulations, the reader is referred to the Executive Summary for a statement of them.

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